Galois Theory, Progress Test 1

February 25, 2020

Please find below a brief report on Progress Test 1.

The test was taken by 30 students. The average final mark was 10 out of 20. Only 12 students received a final mark > 10. Very few students provided precise and complete answers. No marks were assigned to answers without explanations.

Question 1 This question had an average of 6 out of 10. However, there were quite a few answers with very low marks: 10 students received a mark < 5. None of the students managed to get the full 10 marks.

(a) This part was OK on average, although many students provided a (not very precise) argument based on real analysis instead of algebra. A few students seem to believe that if p and q are two coprime integers and $q|p$ then $p/q = \pm 1$.

(b) This part was successfully tackled by most of the students. Marks were assigned as follows: 1/3 to find out that 1 is a root of $\overline{f}$, 1/3 to show that $\overline{f}(x) = (x+1) \cdot \overline{g}(x)$ where $\overline{g}(x) = x^3 + x^2 + 1$, and 1/3 to show that $\overline{g}$ is irreducible over $\mathbb{F}_2$. A couple of students made miscalculations when dividing $\overline{f}$ by $x + 1$. Some students did not explain why $\overline{g}$ is irreducible over $\mathbb{F}_2$, others provided an incomplete explanation of this fact.

(c) Many missing or incomplete answers here. Only 2 students gave an entirely correct answer to this part and explained that, if f factors as $f = g_1 \cdot g_2$ in $\mathbb{Q}[x]$, then by the Gauss Lemma we may assume that $g_1, g_2 \in \mathbb{Z}[x]$. A student believes that the fact that f has no rational roots implies the irreducibility of f over $\mathbb{Q}$.

Question 2 This question had an average of 4 out of 10. There were lots of answers with very low marks: 15 students received a mark < 3. Only 3 students got the full 10 marks.

(a) A worrying number of wrong answers here. Quite a few students believe that, since the roots of $x^5 - 1$ are the 5^{th} roots of unity, $[K : \mathbb{Q}] = 5$. Most of the students provided an incomplete argument to show that $[K : \mathbb{Q}] = 4$. For instance, many students did not mention that the splitting field of $h(x) = x^4+x^3+x^2+x+1$ is $\mathbb{Q}(\xi)$, nor that this field is a degree 4 extension of $\mathbb{Q}$ **since** h is the minimal polynomial of ξ over $\mathbb{Q}$.

(b) Many incomplete answers here. Marks were assigned as follows: 2/4 to find the required polynomial g, 1/4 to solve the quadratic equation and observe that $\alpha > 0$, 1/4 to find the value of $\cos \frac{2\pi i}{5}$. Many students did not understand they needed to use that $h(\xi) = 0$ to find the polynomial g, some of them made some miscalculations and found the wrong one. Some students did not notice that $\alpha > 0$. A student seems to believe that $\exp(\frac{2\pi i}{5}) = \sin \frac{2\pi i}{5} + i \cos \frac{2\pi i}{5}$.

(c) Many incomplete or wrong answers here. Marks were assigned as follows: 2/4 to prove that $G \simeq C_4$, 1/4 to establish that $\mathbb{Q} \subset K$ has only a nontrivial intermediate field F, and 1/4 to say what F is. A worrying number of students concluded that G is isomorphic to the group D_{10}, some students established that $G \simeq V_4$. Many students provided incomplete arguments to show that $G \simeq C_4$. However, most of the students who understood that $G \simeq C_4$ provided a correct description of the fields $\mathbb{Q} \subset F \subset K$.