Galois Theory, Progress Test 2

March 22, 2019

Please find below a brief report on Progress Test 2.

(a) This part was correctly tackled by most of the students. Marks were assigned as follows: 1/5 for proving that f is irreducible, 1/5 for computing $(\alpha+1)^3$, 2/5 for finding the other two roots of f and 1/5 for explaining that f splits over F. The most common argument to prove the irreducibility of f was to show that f has no roots in $\mathbb{F}_3$ (however, most of the students did not explain that such argument can be used since f is a cubic polynomial). Some students did not manage to determine the third root of f (besides α and $\alpha+1$), or got it wrong; none used the properties of the Frobenius to find out that the third root of F is $\alpha-1$ (all the students who got this right did it by guessing and checking).

(b) This part was OK on average. Marks were assigned as follows: 3/5 for computing β^2 and β^3 in the basis $\{1, \alpha, \alpha^2\}$, and 2/5 to find the minimal polynomial of β over $\mathbb{F}_3$. A couple of students made some miscalculations when computing β^3. $\sim 1/3$ of the students did not compute the minimal polynomial of β over $\mathbb{F}_3$, or got it wrong.

(c) Only 6 students gave an entirely correct answer to this part. Marks were assigned as follows: 1/5 to prove that g is irreducible over $\mathbb{F}_3$, 2/5 to determine the possible values of $\varphi(\alpha)$ in the basis $\{1, \beta, \beta^2\}$, and 2/5 to explain there are only three possible field homomorphisms $\varphi\colon F \to F^\star$. Almost everyone proved that the polynomial g is irreducible over F_3 (by checking it has no roots in $\mathbb{F}_3$). A worrying number of students wrote that the possible images of α through φ are the roots of g. Only $\sim 1/3$ of the students understood that the number of field homomorphisms $\varphi\colon F \to F^\star$ equals the number of roots of f in $F^\star$.

(d) Only 3 students gave an entirely correct answer to this part. Marks were assigned as follows: 2/5 to compute the degree of the minimal polynomial h of γ over $\mathbb{F}_3$, 1/5 to explain that this polynomial splits over F, and 2/5 to find the number of monic irreducible cubic polynomials in $\mathbb{F}_3[x]$. The most recurrent argument to show that h is cubic was to apply the tower law to the chain of field extensions $\mathbb{F}_3 \subset \mathbb{F}_3(\gamma) \subset F$. $\sim 1/2$ of the students did not seem to remember that finite extensions of finite fields are normal. Almost all students who found out that there are precisely 8 monic irreducible cubic polynomials in $\mathbb{F}_3[x]$ got the answer by direct computation.