Galois Theory, Progress Test 2

March 20, 2018

Please find below a brief report on Progress Test 2, question by question.

Question 1 (8 points over 20)

- $K \subset L_1 \cap L_2$ is separable:
  This exercise was successfully solved by most of the students. The two most recurrent arguments were: since $L_1 \cap L_2$ is a subfield of L_1, $i)$ $K \subset L_1$ separable implies $K \subset L_1 \cap L_2$ separable (from lectures), or $ii)$ the definition of separability for $L_1 \cap L_2$ is satisfied, as every chain $K \subset F_1 \subset F_2 \subset L_1 \cap L_2$ gives a chain $K \subset F_1 \subset F_2 \subset L_1$. Few students seem to believe to this wrong statement: $K \subset L$ is separable if and only if $\forall\ a \in L$ the minimal polynomial f of a over K has $\deg(f)$ distinct roots lying in L. (In actual fact the roots need to be distinct but it doesn't matter where they are. They could be in some other, bigger, field.)
- $K \subset L_1 \cdot L_2$ is separable:
  $\sim 2/3$ of the students left this exercise blank and, among the ones who tried it, only a couple of them gave an almost correct answer (almost none proved the fact that if $a_1, \ldots, a_m$ are separable over K, then $K(a_1, \ldots, a_m)$ is a separable over K).

Question 2 (12 points over 20)

Q2 (a) A worrying number of students provided a wrong answer for the degree $[\mathbb{F}_q : \mathbb{F}_p]$, and many students wrote the solution $[\mathbb{F}_q : \mathbb{F}_p] = m$ without giving any kind of explanation.

Q2 (b) A lot of incomplete answers here. Some students tried to show that $\mathbb{F}_p \subset \mathbb{F}_q$ is separable and normal by using the definition of normality and separability, but they got lost. A couple of students said that $\mathbb{F}_q$ is normal over $\mathbb{F}_p$ because it is the splitting field of some polynomial in $\mathbb{F}_p[x]$, but they did not write what the polynomial is (or wrote the wrong one).

Q2 (c) A lot of incomplete answers also here. Some students forgot to show that F fixes $\mathbb{F}_p$, while others showed that F is compatible with the product but did not do the same with the sum (compatibility with respect to the sum is a necessary condition for F to be a field homomorphism!). Many students seem not to know that a field homomorphisms is automatically injective (thus, also surjective, in this case).

Q2 (d) $\sim 1/2$ of missing or wrong answers and only a couple of fully correct and complete answers. All the other answers missed to mention/prove some important facts: for instance, some students did not explain that, since the field extension is normal and separable, $|G| = m$; other students did not notice that F in (c) is an element of G; also, most of students use the fact that $\mathrm{ord}_G F = m$ without proving that $F^r \neq id$ if $r < m$.

Q2 (e) $\sim 1/2$ of missing or wrong answers. The remaining students provided a correct argument here.