Galois Theory, Progress Test 1

February 20, 2018

Please find below a brief report on Progress Test 1, question by question.

Question 1 (9 points over 20)

Q1 (i) The criterion was stated correctly by most of the students. Some students got 0.5/1 since they forgot to write which ring the polynomial f in question must belong to or because they missed some conditions the prime p must satisfy. The whole point was deducted when the criterion was given as a necessary condition for a polynomial to be irreducible, when the divisibility conditions holding among p and the coefficients were the wrong ones or when f was required to belong to $\mathbb{Q}[x]$ (the criterion is significant when $f \in A[x],\ A$ UFD!).

Q1 (ii)(a) The exercise was successfully solved by almost all students. Students who wrote the factorisation into irreducibles without justifying the irreducibility of the polynomial $x^4+x^3+x^2+x+1$ got 1/2, while students who didn't even claim that $x^4+x^3+x^2+x+1$ is irreducible got 0.5/2.

Q1 (ii)(b) Nothing to mention. Basically no mistakes here.

Q1 (ii)(c) Also this exercise was correctly tackled by almost all students. The two most recurrent arguments were: i) Eisenstein's criterion with $p = 5$, or ii) a polynomial of degree 3 in $\mathbb{Q}[x]$ is irreducible if and only if it has no rational roots. Some marks were deducted due to inaccuracies in the argument (e.g. $x^3 = 5$ does not imply $x = \sqrt[3]{5}$, or, also, a primitive polynomial $p \in \mathbb{Z}[x]$ of degree 3 which is reducible over $\mathbb{Z}$ must have a linear factor, but not necessarily an integer root - one needs to observe that the polynomial is monic to conclude this).

Q1 (ii)(d) The exercise was OK on average. As in Q1 (ii)(a), students who wrote the factorisation into irreducibles without justifying that $x^3 + x + 1$ is so got 1/2 and students who did not even claim that $x^3 + x + 1$ is irreducible got 0.5/2. A few students left the exercise blank and two students solved the wrong exercise. As in Q1 (ii)(c), some marks were deducted due to inaccuracies in the argument: assuming there exists a rational root a/b of $x^3 + x + 1$, with a and p coprime, a contradiction can be deduced only after excluding the case $a = \pm 1,\ b = \pm 1$, and, again, as in Q1 (ii)(c), having a linear factor in $\mathbb{Z}[x]$ is equivalent to have an integer root only because the polynomial considered is monic.

Question 2 (11 points over 20)

Q2 (a) Almost everyone provided a correct proof here. (A few students do not know the sine and the cosine of $2\pi/3$!!!)

Q2 (b) $\sim 1/4$ of the students did not understand how to use part (a) to find a polynomial with w as a root or left the exercise blank, while practically all the students who understood the hint got the exercise right.
Some students stated that the polynomial $p(x) = w^3 - 3w + 1$ was irreducible without providing a justification ($-0.5/3$), and a few of them did not even mention that p was irreducible to conclude it was the minimal polynomial of w ($-1/3$).

Q2 (c) $\sim 1/3$ of the students provided a wrong solution or left the exercise blank, while a few students provided a correct answer for K_1, but without justifying it ($-1.5/3$) or giving only a partial explanation ($-0.5/3$ or $-1.5/3$). A common mistake was setting $K_1 = \mathbb{Q}(\sqrt{3})$ (with this choice, K_1 is not contained in K!). Further, some students seem not to know the notation for the degree of an extension of fields: if $L \subset K$, one writes $[K : L]$ and not $[L : K]$.

Q2 (d) Lots of missing or incomplete answers here. On average, in order to compute the degree $[K : \mathbb{Q}]$, the tower law was applied correctly, but the fact that $[K : K_1] = 3$ was left without an explanation or with only a partial one. On the other hand, the few who fully computed $[K : \mathbb{Q}]$ used several different proofs, some of them quite nice ones. Regarding the second part, WITH the assumption that $[K : \mathbb{Q}] = 6$, the argument to conclude the irreducibility of ϕ was well explained in general.